扩展卡尔曼滤波

BY JIN

2019-03-13

1 预测更新

设系统在$t, t+1$时刻的状态值有函数关系：

$$x_{t+1} = f(x_t) + w_t, \quad w_t \sim \mathcal{N}(0, Q_t) \tag{1}$$

自然地，有：

$$\hat{x}_{t+1} = f(\hat{x}_t) \tag{2}$$

其误差

$$\begin{aligned}\varepsilon_{t+1} &= x_{t+1} - \hat{x}_{t+1} \\ &= f(x_t) + w_t - f(\hat{x}_t) \\ &= f(\hat{x}_t) + f'(\hat{x}_t)(x_t - \hat{x}_t) + O((x_t - \hat{x}_t)^2) + w_t - f(\hat{x}_t) \\ &= f'(\hat{x}_t)(x_t - \hat{x}_t) + w_t + O((x_t - \hat{x}_t)^2)\end{aligned} \tag{3}$$

忽略高阶无穷小，求协方差矩阵得：

$$\begin{aligned}\hat{P}_{t+1} &= E(\varepsilon_{t+1}\varepsilon_{t+1}^T) \\ &= E((f'(\hat{x}_t)(x_t - \hat{x}_t))(f'(\hat{x}_t)(x_t - \hat{x}_t))^T) + E(w_t w_t^T) + O((x_t - \hat{x}_t)^3) \\ &\approx f'(\hat{x}_t)E((x_t - \hat{x}_t)(x_t - \hat{x}_t)^T)f'(\hat{x}_t)^T + E(w_t w_t^T) \\ &= F_t \hat{P}_t F_t^T + Q_t\end{aligned} \tag{4}$$

其中$F_t = f'(\hat{x}_t), Q_t$为随机误差w_t的方差。

2 测量更新

设系统状态x与观测值z之前有函数关系：

$$z = h(x) + v, x \sim \mathcal{N}(\mu, P), v \sim \mathcal{N}(0, R) \tag{5}$$

现有此刻系统状态的估计值x_0，则对观测值的估计为：

$$z_0 = h(x_0) \tag{6}$$

又有此刻的观测值z，欲求x。通过泰勒展开线性化式(5)：

$$z = z_0 + h'(x_0)\,(x - x_0) + O((x - x_0)^2) + v \tag{7}$$

令$\delta z = z - z_0, \delta x = x - x_0$，且令$H = h'(x_0) = \frac{dz}{dx}|_{x = x_0}$为$h: x \to z$的雅可比矩阵，则有：

$$\delta z \approx H\delta x + v, \delta x \sim \mathcal{N}(0, P), v \sim \mathcal{N}(0, R) \tag{8}$$

解出Kalman增益$K = PH^T(HPH^T + R)^{-1}$，则δx的最优后验估计为：

$$\widehat{\delta x} = 0 + K(\delta z - 0) = K\delta z \tag{9}$$

相应地：

$$\hat{x} = x_0 + \widehat{\delta x} = x_0 + K\,(z - h(x_0)) \tag{10}$$

由于x与δx的方差相同，更新x的方差估计为：

$$\hat{P} = (I - KH)P \tag{11}$$